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A comparison of methods of strain determination in rocks from southwest Dyfed (Pembrokeshire) and adjacent areas

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Abstract—A variety of methods devised in the last twelve years for measuring two-dimensional strain ratios have been applied to the same deformed oolitic limestone from southwest Dyfed. Their relative merits are discussed. A graphical method for studying relative locations of marker objects is described and its application to a variety of rocks illustrated. It is shown to have theoretical and practical advantages, while giving values of two-dimensional strain ratios comparable with those of other methods.

INTRODUCTION

FRY (in press, a) describes a method of measuring natural strain in rocks from the relative locations of marker objects. It involves the creation of an 'autocovariance' or 'all-object-separations' plot, on which the distance and direction from each marker to each other marker in the rock are recorded by a point at that distance and direction from the origin. From an isotropic distribution such a plot has a concentric circular pattern. From a deformed distribution which was initially isotropic, the pattern is concentric elliptical, with the ellipse axial ratio and orientation corresponding to the strain ellipse of the deformation.

Fry (in press, a) is largely concerned with theoretically justifying his method but he also suggests that the best procedure is numerical handling of marker coordinates. Hanna, as part of a study of regional rock deformation, has used a manual graphical procedure and has shown it to have practical advantages not foreseen by Fry (in press, a). This is the subject of the present paper.

TECHNIQUES USED IN THE COMPARISON

Many techniques for measuring the finite strain of rocks have been documented in the last twelve years. These generally involve the determination of two-dimensional strain on planar sample sections through a rock, with the intention that three such planes may be combined to yield the three-dimensional strain. This paper briefly discusses several methods for determining two-dimensional strain (see Fig. 2 and Table 1). These methods are later referred to informally or by our letters A, B, C, etc. References are only given fully in this section.

Method A—Method 1 *of Ramsay* (1967, *p.* 193) *and Cloos* (1947)

This method uses as strain markers objects which were approximately circular in their initial state. Direct measurement is made of the lengths of the principal axes of the strained object ellipses. Either the arithmetic mean of the individual observations is calculated or the data are plotted as a graph of the short axis length vs the long axis length. The slope of a line through these points is taken to be the ratio of the principal strains within the particular section.

Method B—Method 3 *of Ramsay* (1967, *pp.* 195–197), *'centre-to-centre' method*

This method assumes that the length of a line from the centre of an object to the centre of one of its nearest neighbours is initially statistically independent of the line's direction. In no other respect is the initial distribution necessarily assumed to be isotropic. The method considers only the relative displacement of the centres of adjacent objects by the deformation, and does not rely on elliptical shapes of strained objects to indicate the strain ellipse. It is useful for rocks composed of packed objects (ooids in Ramsay 1967) which have had the regularity of their external shapes destroyed by pressure solution. The strain ratio is estimated from a graph of length (d) against orientation (α), relative to a common azimuth, of every measured line between centres of adjacent objects.

Method C—Dunnet (1969) *and Dunnet & Siddans* (1971), Rf/φ *method*

This method assumes a homogeneous deformation of initially elliptical marker objects with their matrix. Initially randomly variable in both ellipticity and orientation, both these features are changed by the deformation. A plot is made of log Rf, where Rf is the final ellipse ratio, against φ, the final long axis orientation, of each ellipse. These parameters depend on five variables (Dunnet 1969, p. 117); initial particle shape, initial particle orientation, strain intensity, strain orientation and the degree of ductility contrast between particles and the entire particle/matrix system. The method is extended by Dunnet & Siddans (1971) to include planar, linear and imbricate sedimentary fabrics. They outline two computer programs, one of which constructs standard Rf/φ curves and '50% of data' curves for different Rf values. The other programme (our method F) is a procedure for determining the strain ratio and orientation of sections through certain homogeneously deformed planar and semiplanar fabrics by re-establishing initial symmetry.

Method D—Fry (in press, a)

This method assumes that before deformation the locations of the centres of intersections of marker objects by a sample plane would on any sample plane have an isotropic, but non-Poisson, distribution.

If the position-marking objects behave passively during deformation, the ellipticity and orientation of an 'autocovariance' or 'all-object-separations' plot correspond to those of the strain ellipse. Fry (in press, a) suggests a numerical approach, but the following purely manual plotting procedure has been adopted in this study. Place a sheet of tracing paper, on which a series of parallel reference lines have been drawn, over the sample and mark the centre of every marked object within the chosen sample field. Take a second sheet of tracing paper with a centre point marked on it and a set of marked lines which can be kept aligned parallel to those on the first sheet. Place the central point of the second sheet on one marked object position on the first, and mark on the second sheet all the positions of points on the first. Now place the central point of the second sheet on a different point on the first, and again mark on the second sheet all positions of points on the first. Continue accumulating points by this procedure, maintaining the same orientation, until all points on the first sheet have been used as centres on the second. The resulting plot on the second sheet has an ellipticity of the same ratio and orientation as the strain ellipse. The long and short axial lengths and orientations are measured directly with ruler and protractor. Theoretical limitations, with a discussion of edge effects and other complications, are given by Fry (in press, a).

Method E—Shimamoto & Ikeda (1976)

Shimamoto & Ikeda (1976) assume all objects to be initially ellipsoidal and to comprise an initially isotropic population. Using as data the dimensions and orientations Rf and φ (Dunnet 1969) of a large number of object intersections, the descriptive parameters of an overall ellipse for the sample plane can be numerically determined. This ellipse should converge in the limit precisely to the strain ellipse, provided a correct numerical procedure is adopted. In use by a number of workers in the past as a method for measuring strain, Shimamoto & Ikeda appear to be the first to have published the procedure fully, adopting a description of it in terms of shape matrices. The existing University College of Swansea computer programme for this procedure was used for our study.

Method F—Dunnet & Siddàns (1971), numerical version of Rf/φ

This computation, using a programme similar to their "STRANE" (p. 322) checks symmetry about "$\varphi = 0$" and "50% of data" curves for tested strain values, assigning the optimum tested value of strain.

Method G—Mimran (1976) Density distribution technique

This technique is based on the principle that the longer the dimension of any strain marker object in a given orientation, the higher the probability of its being intersected by sample planes perpendicular to this orientation. Provided that a population of marker objects is initially isotropic, the density of intersections of objects by a plane in the strained state is a measure of the perpendicular dimension of the strain ellipsoid. Mimran's development of the technique involves counting objects on three perpendicular sample planes, but his method of constructing three two-dimensional strain ellipses from such data is theoretically unsound (Fry in press, b). A correct use of intersection density data does not permit direct specification of two-dimensional strains for direct comparison with the results of the other methods used in this paper.

COMPARISON OF RESULTS

The methods outlined above have been applied to three mutually perpendicular plane sections of an oolitic limestone sample from West Angle Bay, southwest Dyfed (formerly Pembrokeshire). The results are summarized in Table 1. Although judged subjectively to be principal planes of the strain before sectioning, there is no way to corroborate or refute such a relationship independently of the determined two-dimensional strain ratios of the sample sections. Therefore comparison of the values for the different sections on the assumption of principal planes is not a valid test of the methods, and we do not here refer these orientations to know principal directions. That the product of the three strain ratios is not unity indicates that these are not principal planes, and a forthcoming paper (Fry in press, b),

Table 1. Measured strain ratios on three studied sections through an oolitic limestone from West Angle Bay, Dyfed for the methods discussed. The sections 1, 2 and 3 were chosen to lie approximately in XY, YZ and XZ, respectively

Sample plane:	1	2	3
Method: A: Ramsay (1967, p. 193) long/short	1.74	2.52	3.31
B: (1967, p. 195) centre-to-centre	1.78	2.44	3.50
C: Dunnet & Siddans (1971) *Rf*/φ	1.65	2.40	3.25
D: Fry (1979) our version	1.66	2.42	3.27
E: Shimamoto & Ikeda (1976) (numerical)	1.68	2.41	3.26
F: Dunnet & Siddans (1971) (numerical)	1.69	2.42	3.24
G: Mimran (1976) density distribution	1.23	1.42	1.76

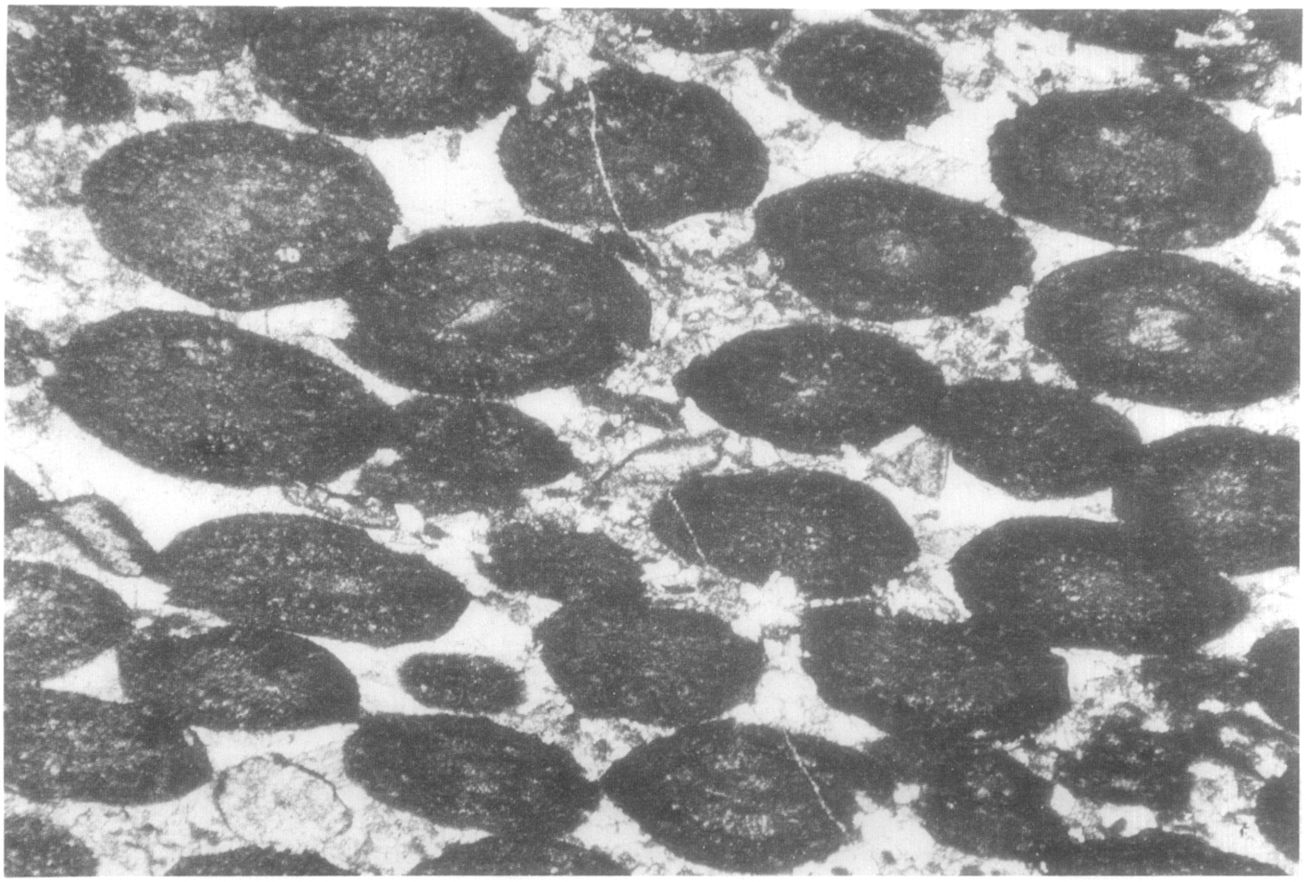

Fig. 1. Photomicrograph of the approximately YZ section of the oolitic limestone sample for which results of different methods of estimating strain ratio are given in Table 1 and illustrated in Fig. 2. Average ooid length 0.6 mm.

which discusses errors due to misjudging principal planes, would here indicate misorientations of at least 10°. This matter is not considered further because it is methods of two-dimensional strain ratio measurement which are the subject of this paper, the forthcoming paper discusses three-dimensional strain determination and errors. Internal consistency of the data for each plane is our only judge of the accuracy of the particular two-dimensional strain ratios.

General comments on the methods

It should be noted that methods A, C, E and F rely on the measurement of elliptical sections of objects assumed also to have been elliptical in their initial state. If this assumption applies to a particular rock, valuable information would be lost by not using one of these methods. On the other hand, these methods determine strain of objects, which may not correspond to the overall rock strain. Methods B, D and G do not rely on individual object shapes and are therefore more generally applicable. The similarity of results for our sample indicates the homogeneity of the deformation of this particular rock, which is illustrated in Fig. 1.

Even in an oolitic limestone, object shape may be difficult to use. Difficulties noted by Cloos (1947) and cited by Ramsay (1967, p. 193) are: (a) frayed ooid ends or pressure dissolved sides; (b) unidentifiable ooid ends at high strains and (c) difficulty in locating principal ellipse axes when strain is slight.

Note also that it is only methods C, D and F (our method and the Rf/φ method) which provide information about the validity of the assumptions on which they are based.

The data plots for the graphical methods A–D for each of the sample planes are shown in Fig. 2.

Remarks on each method

Method A. The long-to-short method is often quick and easy to plot, but problems of definition of a best-line offset these advantages. Whatever definition is used, this line is more reliable if it is calculated. Its slope can be shown on theoretical grounds to be an overestimate of the strain ratio by an amount which is unknown and is dependent upon dispersion of original shapes from truly circular, and upon any correlation of either this dispersion or of competence contrast with grain size. These latter variations receive different weightings in the definitions given below, with values shown for this sample:

	Plane 1	Plane 2	Plane 3
$\Sigma X/\Sigma Y$	1.739	2.55	3.40
$\dfrac{\Sigma(X/Y)^{\frac{1}{2}}}{\Sigma(Y/X)^{\frac{1}{2}}}$	1.754	2.57	3.41
$\left(\dfrac{\Sigma X/Y}{\Sigma Y/X}\right)^{\frac{1}{2}}$	1.741	2.55	3.40
'Eye-balled'	1.740	2.52	3.31

Method B. The 'centre-to-centre' method is slow and laborious, and it is not always clear how best to define the maximum and minimum distances from the plot (see discussion of method D).

Method C. The Rf/φ methods are slow but can provide interpretable information on initial fabrics, and on the relative orientation of strain to bedding and cleavage directions, in addition to giving strain values. These relationships give these methods considerable advantage (shared by our method D) in the unravelling of large-scale folds or other structures.

The number of objects required varies. Estimates include: 40 ooids or 60–100 pebbles in conglomerates (Dunnet & Siddans 1971); 30 ooids (Tan 1976, p. 167) and 20 ooids for strain ratios of 4 : 1 or greater in this study. This study was made using the D-mac digitizer on-line to a Digico Micro 16V minicomputer at Leeds University (Siddans 1976).

Method D. This version of the method described by Fry (in press, a) is much quicker than any of the other methods because the data are accumulated graphically as the final plot without being first rendered in numerical form. Definition of the ellipse ratio appears to lack precision, but in practice the reproducibility is found to be good and values obtained compare well with other methods. This applies particularly here because of the very sharp cut-off to neighbour distances caused by touching of well-sorted spherical objects (ooids) in the initial state. These same maximum and minimum distances to the same cut-off are also the best definition of the ellipse ratio when data is handled using Ramsay's 'centre-to-centre' method (our method B). On Ramsay's plot the cut-off should be sinusoidal, while by this method it is elliptical. In practice it is easier to visualise the ellipse form. Thus our method is of advantage in both the preparation and interpretation of the graphical form. Note that the methods are only directly comparable when it is, as with this oolitic sample, the initially nearest-neighbour cut-off which is being measured (see later discussion).

Method E. The time taken by the numerical methods (E and F) is only that required to convert the dimensions and orientations Rf and φ of object intersections into digital form. An important limitation to most such calculations is that although the results are correct only for an initially isotropic population, this assumption is not tested by the method. Such a limitation does not apply to the numerical method of Matthews *et al.* (1974) which we have not tried. In as much as their method tests for symmetry of anisotropic configurations it is equivalent to method F with a simpler mathematical base.

Method F. Siddans' (1976) version of the Dunnet & Siddans (1971) calculation is complicated but does test for initial isotropy. However, if isotropy is established the methods of Matthews *et al.* (1974) and method E are superior.

Method G. In view of the incorrect development by Mimran (1976) of his density distribution technique, and its inability to give true two-dimensional strain ellipses other than by working back from three-dimensional strain (Fry in press, b), we include values here only to show that they are out-of-line with other methods.

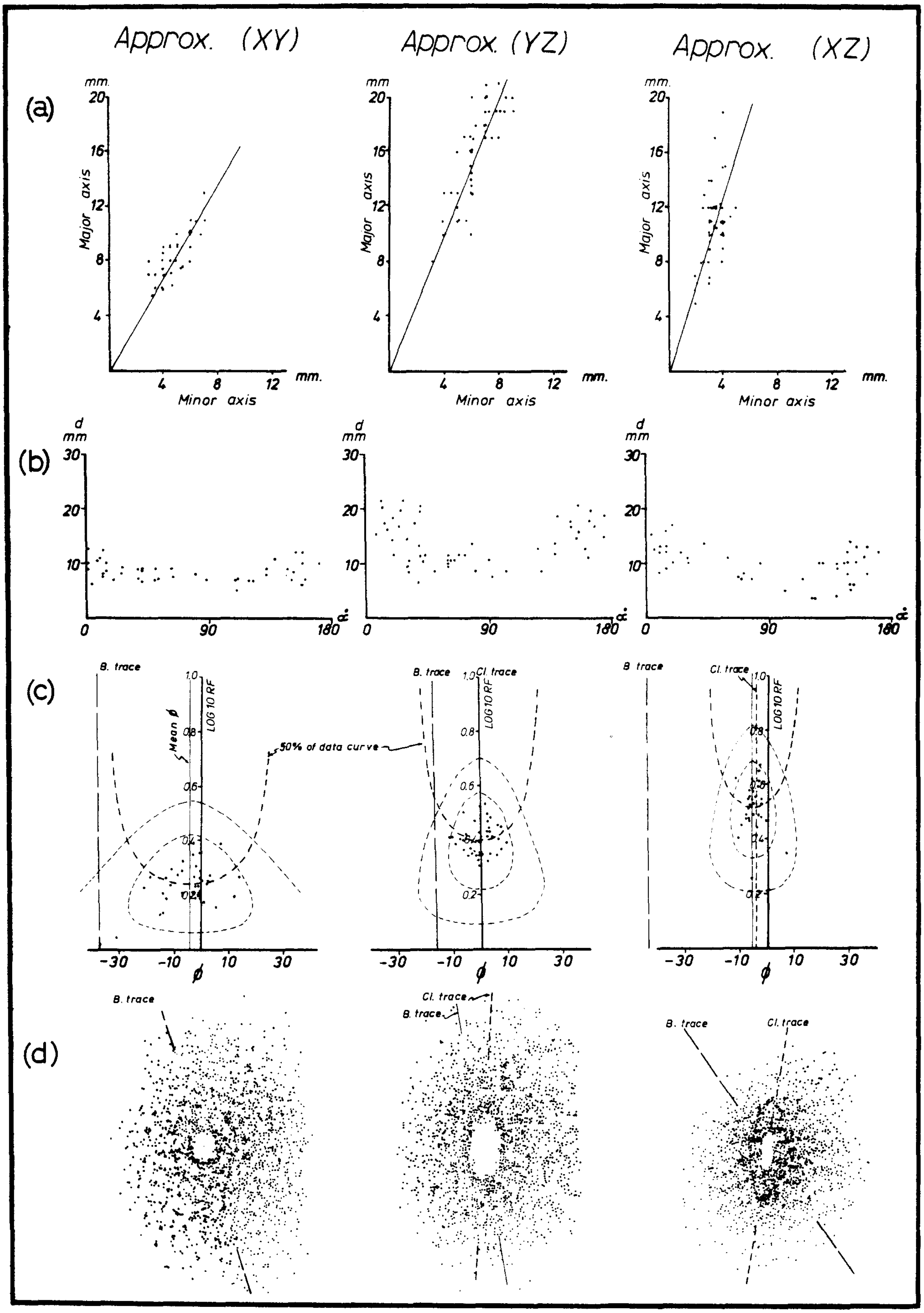

Fig. 2. Plots for each of the three sample planes of the oolitic limestone specimen from West Angle Bay, Dyfed, for the four graphical methods discussed. Letters (a)–(d) refer to methods A–D in the text.

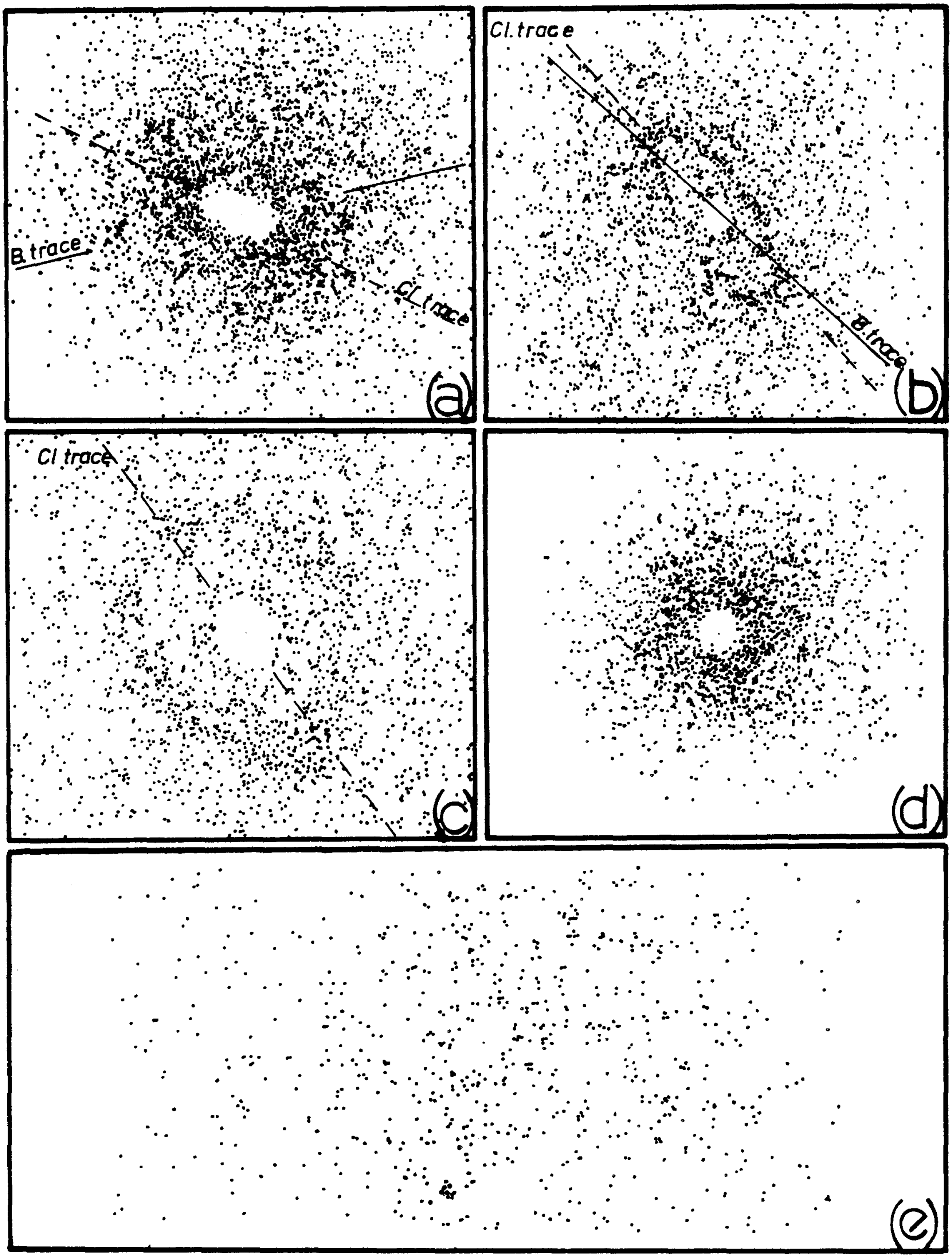

Fig. 3. Plots for the rock samples mentioned in the further discussion of our method. (a) Oolitic Lower Carboniferous limestone from Caswell Bay, Gower. (b) Reduction spots in Old Red Sandstone mudstones, Ballycotton, Co. Cork. (c) Ferruginous rhizoid-fill nodules in Coal Measures sandstone, Amroth, Dyfed. (d) Calcareous concretions in Old Red Sandstone mudstones, Llanstephan, Dyfed. (e) Pyrite aggregates in Carboniferous slates, Bantry, Co. Cork.

ADVANTAGES OF THE MANUAL GRAPHICAL METHOD

The manual plotting version of the method of Fry (in press, a) has been applied to many types of marker objects. Those reported here from South Wales are ooids in oolitic Lower Carboniferous limestones from West Angle Bay, Dyfed (Fig. 2) and Caswell Bay, Gower (Fig. 3a), rhizoid-fills in Westphalian sand-stone from Amroth, Dyfed (Fig. 3c) and calcareous concretions in Old Red Sandstone mudstones from Llanstephan, Dyfed (Fig. 3d). Also illustrated are reduction spots in mudstones (Fig. 3b) and pyrite aggregates in slates (Fig. 3e) from localities in Ireland.

Acetate peels were made from etched cut surfaces of the oolitic limestones and then projected onto a ground glass screen. The other rocks were photographed in the field and a transparency projected in the same manner. Thin sections may be similarly treated, as the only thickness effect is a modification of the object distribution's density but not its anisotropy. We see no reason why, given a flat rock surface with a suitable density of marker objects, the method should not be used directly in the field.

The concentric similar elliptical pattern may have some very clear initially circular feature, such as a cut-off due to ooid size. This is found to be clear and easily measurable with a sample size of about 60 ooids, taking about half an hour to complete. Fry (in press, a) mentioned that for poorly-defined patterns a sample of several hundred objects may be necessary, and computer handling of digitized coordinates would then become worthwhile. In practice, this study has shown that the sample size need only be raised to about 140 for concretions, and the time taken never exceeded 1 h. Fry (in press, a) mentioned that Ramsay's 'centre-to-centre' method (method B) gives greater accuracy for small sample size. We find that this is not true for ooids, and that it is quicker to handle a large sample by our method than a selected small part of it by Ramsay's method.

Some distributions of markers give a plot which has a gradationally edged girdle of higher density of points instead of a sharp cut-off. Fry (in press, a) states as the main advantage of his method that it produces true strain ratios from such distributions, whereas nearest-neighbour methods give false results. This study confirms this advantage, Fig. 3(b) being such a plot.

Figures 3(c) and 2(d) are of particular interest in having both an inner-neighbour cut-off ellipse and a denser outer ellipse. In general, these may be different in ellipticity and orientation, the cut-off showing the average strain of marker object shapes that method B would determine, while the outer dense girdle shows the overall strain of the rock. Their similarity in Fig. 2(d) is confirmation of the homogeneity of the strain of this sample.

Figure 3(c) deserves particular attention, although the denser ellipse is not accurately located by such a small number of plotted points. The strain of this rock is by pressure solution striping (Beach & King 1978). The preferred spacing of markers is greater than that of the stripes, so that the ellipse of Fig. 3(c) portrays the overall rock strain resulting from the pressure solution. We are not aware of any other method that can determine the strain of this rock.

Figure 3 illustrates that this method is able to show the relationship of the traces of bedding and cleavage to ellipse girdle. Figure 3 also shows two samples for which the method fails to give a value for the strain. Figure 3(d) shows an isotropic pattern of centres of packed competent calcareous nodules in a deformed rock, in which we deduce that sliding at nodule boundaries has accommodated the strain. Figure 3(e) shows a pattern of centres of pyrite aggregates sufficiently close to Poisson (see Fry in press, a) such that no information on strain is forthcoming, whatever the strain. This puts our method at no disadvantage, as all methods fail on these two samples.

CONCLUSIONS

We conclude that our method and the Rf/φ methods are the best of those tested, as information is directly available about the relationship of strain to other features, and about the validity of assumptions in the methods. Our method is the quicker. It does not employ all usable information, however it is applicable to many rock types for which Rf/φ methods cannot be used, and in some cases for which there is no other method. We would recommend trial of our method even if Rf/φ is being used, as there is a possibility of showing discrepancies between marker object strains and overall rock strain, whether as a result of normal ductility contrast or of differences in proportion of volume loss between objects and matrix.

A critical comparison of manual and numerical versions of Fry's method will be the subject of a forthcoming paper.

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